

Noise-assisted traffic of spikes through neuronal junctions

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(Received 4 December 2000; accepted 17 April 2001; published 31 August 2001)

The presence of noise, i.e., random fluctuations, in the nervous system raises at least two different questions. First, is there a *constructive* role noise can play for signal transmission in a neuron channel? Second, what is the advantage of the power spectra observed for the neuron activity to be shaped like $1/f^k$? To address these questions a simple stochastic model for a junction in neural spike traffic channels is presented. Side channel traffic enters main channel traffic depending on the spike rate of the latter one. The main channel traffic itself is triggered by various noise processes such as Poissonian noise or the zero crossings of Gaussian $1/f^k$ noise whereas the variation of the exponent k gives rise to a maximum of the overall traffic efficiency. It is shown that the colored noise is superior to the Poissonian and, in certain cases, to deterministic, periodically ordered traffic. Further, if this periodicity itself is modulated by Gaussian noise with different spectral exponents k , then such modulation can lead to noise-assisted traffic as well. The model presented can also be used to consider car traffic at a junction between a main and a side road and to show how randomness can enhance the traffic efficiency in a network. © 2001 American Institute of Physics. [DOI: 10.1063/1.1379308]

Recent research shows that random fluctuations need not always be destructive in nature by degrading system performance; on the contrary, in nonlinear systems they can support structures, synchronize different processes, or enhance the quality of signal transmission. The latter possibility is investigated in this article. A simple model of a junction in a neuronal structure (as well as in a road structure) is studied using various types of noise to generate the patterns of neural spike traffic. The types of noise differ in their statistical properties including their power spectra. Variation in the noise spectrum is particularly interesting, because the power spectra $S(f)$ in neuronal recordings (as well as in car traffic recordings) have been observed to be shaped like $1/f^k$, where f is the frequency and k is an exponent close to 1. Applying different types of noise to modulate the dynamics of our model we find that random fluctuations are indeed able to enhance signal transmission. We show that the noise with a $1/f$ -like spectrum, briefly called $1/f$ noise, is superior to the other kinds of noise studied here.

Numerous nonlinear systems exhibit SR and are often called “stochastic resonators.”^{4,7} A proper tuning of the input noise intensity is needed to optimize the transfer of a signal through the stochastic resonator. One of the most important research directions of today’s SR studies is the investigation of signal transfer in chemical¹¹ and biological^{2,8} systems, however, there are many other fields such as threshold devices,^{4,7,10} superconducting quantum interference devices,¹³ etc., where SR has been studied. The phenomenon discussed in the present paper is a new type of stochastic resonance effect, where the optimal tuning concerns the shape of the power density spectrum rather than the intensity of the input noise.

Probably, every car driver has experienced a beneficial effect of randomness. In a certain crowded traffic situation, it is not possible to get into a major road from a lower priority road, if the cars are passing the junction periodically. The gaps between subsequent cars on the main road are not large enough for a waiting car to get into that gap and to accelerate to the required speed. Only a larger gap, which usually occurs randomly in the car flow, makes it possible to enter. Therefore, noise can be beneficial for the traffic in certain cases. Similar problems of car traffic including the problem of the gap have been extensively studied in the literature in the last three decades.^{15–17}

Discussed from the viewpoint of statistical physics are traffic jams,^{18–20} time series of single-vehicle data,²¹ and stabilization of traffic flow due to interaction.²²

However, the questions of an optimal noise and of a possible stochastic resonance have not been addressed yet according to our literature search. In this paper, we shall investigate the effect of various noise processes on the traffic

I. INTRODUCTION

Noise-induced order¹ and noise-assisted signal transfer in nonlinear systems are hot topics of today’s physics of complex systems. Important examples are stochastic resonance (SR) phenomena in various physical and biological systems,^{2–14} which have recently attracted a strong interest.

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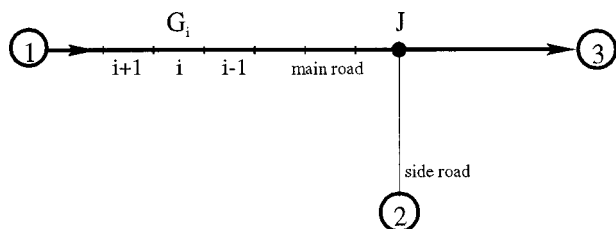


FIG. 1. The model: Main channel traffic from (1) to (3) reaches the junction J . Side channel traffic is waiting at 2 to enter the main channel depending on the i th gap size G_i .

and we shall show that there is an apparent stochastic resonance phenomenon, which concerns the shape of the noise spectrum.

II. THE MODEL

The present model, which has previously been discussed in Refs. 23 and 24, describes neural spike traffic in neuron channels, but can, because of its generality, also be used to describe car traffic on a highway.

The model considers a single-laned one way main road (channel) with a defined distribution of errant cars (spikes). At a junction J a side road is assumed with an infinite number of cars waiting to enter the main road traffic depending on the gap size G_i between two consecutive main road cars. The number N_i of side road cars entering the main road shall be given by the integer function $\text{int}(\cdot)$ of the ratio between the size of the i th gap and a minimal gap size G_0 , where just a single car could enter (see Fig. 1),

$$N_i = \text{int}\left(\frac{G_i}{G_0}\right). \quad (1)$$

The function $\text{int}(\cdot)$ simply reduces the number to the nearest integer value, i.e., truncates the digits behind the decimal place.

To avoid car accidents a minimal gap value could be introduced assuming that the distances G_i and G_0 are measured in conventional length units. Alternatively one can measure G discretely in unit lengths of a car.

To measure the efficiency of the overall traffic (main and side roads) the geometric mean $\nu = \sqrt{\nu_{1,3}\nu_{2,3}}$ has been used, where $\nu_{1,3}$ denotes the mean rate of main road traffic before the junction and $\nu_{2,3}$ the mean rate of side road cars entering the main road, respectively. The geometric mean yields small values even when only one of the traffic channels performs badly. In particular, zero geometric mean would account for the important situation where information flow through one of the channels is blocked completely. It is clear that the arithmetic mean (as a measure of efficiency) does not have this advantage. The information about traffic efficiency can also be supplied by a $\nu_{2,3}(\nu_{1,3})$ plot.

To make the model relevant to neural spike traffic one considers the roads to be neuron channels. As a simplification one could consider both channels to have equal priority. The neuron transfers the spike coming from channel 1 or 2, if the time since the last transferred spike is greater than G_0 , where G_0 represents the refractory time of the neuronal junction.

In this model, the frequency of transferred spikes depends on the statistics of spike generation; moreover, the spikes can be lost. Both the output spike frequency and the probability of spike loss influence the efficiency of information transfer, therefore, the question arises about the kind of spike statistics that provides the best efficiency of information transfer. Since neuronal spikes can be lost in contradiction to cars, the spike transfer probability and number of output spikes are the relevant measures for the neuronal system.

With this model we have investigated three different classes of point processes controlling the generation of the spikes (respective car locations): Poisson process, a case with noise but without memory effects; zero crossing events of colored noise processes with $1/f^k$ -shaped spectra and periodic traffic, a case with no noise. Further on the periodic case has been modified by modulation of the phase with Gaussian noise. Again the $1/f^k$ spectrum of the noise has been varied by tuning the parameter k .

III. NOISE TRIGGERED INPUT

A. Poissonian process

The time moments when cars on the main road pass the junction shall be generated by a Poisson process with rate $\nu_{1,3}^{\text{Pois}}$. The probability to find exactly M events (cars passing the junction) during time interval τ is described by

$$P_M(\tau) = \frac{(\nu_{1,3}^{\text{Pois}} \tau)^M}{M!} \exp(-\nu_{1,3}^{\text{Pois}} \tau). \quad (2)$$

To calculate the relationship between mean rates $\nu_{2,3}$ and $\nu_{1,3}$, we define the moment of a car passing the junction as $t_0 = 0$, and introduce two (positive) observation times t_1 and t_2 , such that $t_1 < t_2$. From Eq. (2) it follows that the probability to have events (one or more) during time interval $t_1 < t < t_2$ if there were no events at $t < t_1$ is

$$P_{M \geq 1, t_1, t_2} = P_0(t_1) - P_0(t_2) = \exp(-\nu_{1,3}^{\text{Pois}} t_1) - \exp(-\nu_{1,3}^{\text{Pois}} t_2). \quad (3)$$

In our model this equation gives the probability of accommodating exactly one car from the junction if we put $t_1 = G_0$ and $t_2 = 2G_0$ (one interval of duration G_0 is "clean," but not two). The probability of accommodating exactly two cars is obtained from Eq. (3) if $t_1 = 2G_0$ and $t_2 = 3G_0$ (two intervals of duration G_0 are clean, but not three). And so on. The mean number of cars accommodated from the junction per single interval between cars in the main road is given by the sum of these probabilities weighted by the corresponding car numbers

$$\begin{aligned} \langle N \rangle &= \sum_{n=1}^{\infty} n [\exp(-\nu_{1,3}^{\text{Pois}} G_0 n) - \exp(-\nu_{1,3}^{\text{Pois}} G_0 (n+1))] \\ &= \frac{\exp(-\nu_{1,3}^{\text{Pois}} G_0)}{1 - \exp(-\nu_{1,3}^{\text{Pois}} G_0)}. \end{aligned} \quad (4)$$

Thus, the average rate of entering from the junction is

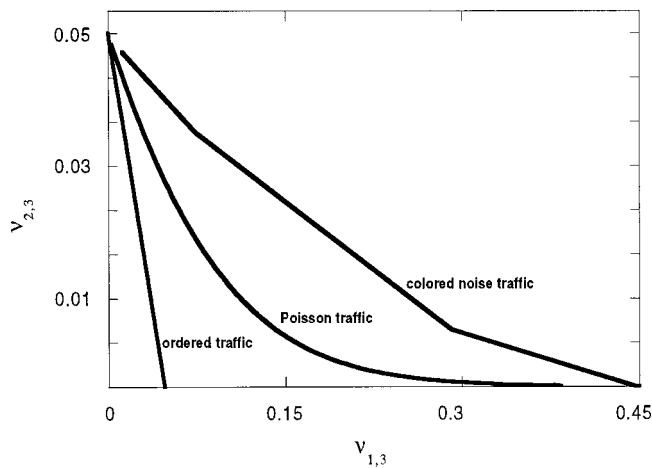


FIG. 2. Superiority of colored ($1/f^k$ generated) traffic over Poissonian and an example for a periodic process.

$$\nu_{2,3}^{\text{Pois}} = \nu_{1,3}^{\text{Pois}} \langle N \rangle = \frac{\nu_{1,3}^{\text{Pois}} \exp(-\nu_{1,3}^{\text{Pois}} G_0)}{1 - \exp(-\nu_{1,3}^{\text{Pois}} G_0)}. \quad (5)$$

Note that this analytical result is exact.

B. Colored noise

Now, consider the case when the point process describing the car occurrence on the main road is generated by the zero crossing events of a Gaussian $1/f^k$ noise. When $k > 0$, this noise has a long-range memory. Due to the experimental evidence of occurrence of $1/f^k$ -like noise processes in car traffic²⁵, it is tempting to apply this kind of noise ($0 < k < 2$) to generate the car occurrence. The mean zero crossing rate of a Gaussian noise process is described by the Rice formula²⁶

$$\nu_{1,3}^{\text{color}} = 2 \frac{\sqrt{(\int_0^\infty f^2 S(f) df)}}{\sqrt{(\int_0^\infty S(f) df)}}, \quad (6)$$

where f is the frequency and $S(f)$ is the power density spectrum of the noise. The value of $\nu_{2,3}$ cannot be calculated analytically because the time distribution of the zero crossing events has been an unsolved problem since 1945.^{26–28}

To compare the different cases of traffic processes, we carried out computer simulations. The length of the simulation and the point processes describing the car occurrence on the main road were 32 768 (2^{15}) and the minimal gap size G_0 was 20. In comparison with a practical highway traffic situation, where the mean distance between the cars is 100 m, the total process length corresponds to the main traffic road of 160 km. In Fig. 2, $\nu_{2,3}$ vs $\nu_{1,3}$ is shown. The results for Poissonian traffic turned out to be in excellent agreement with the predictions of Eq. (5) and can be regarded as a test of the simulation accuracy. The frequency range of integration relevant in Eq. (6) was determined by the simulation length, so the colored noise traffic was solely controlled by the spectral exponent k . For the results shown in Fig. 2, the range 0–2 was used for k . An example of an ordered traffic with the strictly periodic fragments of $G_i \leq G_0$ and regular interruptions with a large G_i to account for $\nu_{1,3} < 1/G_0$ is also

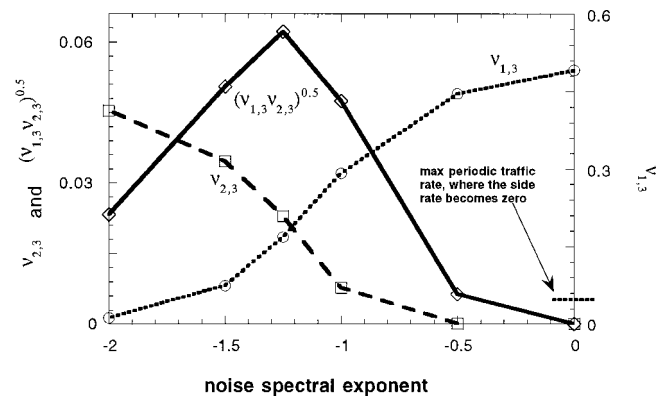


FIG. 3. Stochastic resonance peak in the car traffic efficiency $\nu = \sqrt{\nu_{1,3} \nu_{2,3}}$ (solid line) vs the spectral exponent of the colored noise traffic.

given here. It is obvious that this example of periodic traffic gives the smallest $\nu_{2,3}$ and the colored noise traffic gives the largest $\nu_{2,3}$, especially for medium and high $\nu_{1,3}$. It is also apparent that a better compromise between the $\nu_{1,3}$ and $\nu_{2,3}$ is provided by the colored noise traffic compared to the Poissonian case.

In the case of the colored noise generated traffic, the most interesting result is a new kind of SR, spectral stochastic resonance (SSR), which demonstrates the existence of an optimal spectral shape for the highest traffic efficiency, as shown in Fig. 3. This new kind of stochastic resonance is similar to the classical effects in the sense that the noise driving is needed to get the optimal performance of the system. On the other hand, instead of the noise intensity, the spectral shape (as described by k) is the SR tuning parameter, which optimizes the performance. Note, that usually, the spectral shape is more related to resonance effects in physics than intensity.

Musha and Higuchi²⁵ reported a $1/f$ -like noise in the traffic current of cars on highways, so, it is particularly interesting that the optimal traffic in our model is also found around $k = 1$.

There are also nontrivial features for the neuron traffic model when it is driven by $1/f^k$ noise generated spikes. This time, the introduction of the overall traffic efficiency, as the geometrical mean of the two traffic rates, is not necessary. The mean frequency of the outgoing spikes is a good measure of the variations in the upper limit of information transfer rate through the system. Computer simulations were done in a similar way and with similar conditions as described above. The upper frequency cutoff of the spectrum was chosen as 6000 (compare with the sampling frequency of 32 000), which is equivalent to a refractory time of 5.3, characterizing spike statistics of the sources of spikes. The refractory time of the junction-neuron, which corresponds to the minimal gap size in the car traffic model, was chosen as 10. In Fig. 4, the mean rate of transferred spikes and the probability of spike transfer are shown. The rate of transferred spikes characterizes the highest meaningful bandwidth of information transfer, due to Shannon's sampling theory. Therefore, this quantity is directly related to the information transfer rate.

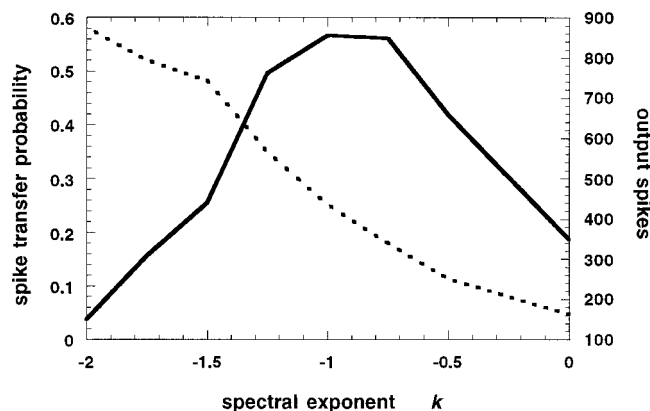


FIG. 4. Stochastic resonance peak in the frequency of transferred neural spikes (solid line) vs the spectral exponent of the colored noise traffic. The dashed line shows the probability of spike loss.

Here we can also observe a well-pronounced SSR around $k = 1$. This fact is in an interesting coincidence with the general occurrence of $1/f$ -like noise phenomena frequently reported in neural activity.^{28–31} The other quantity, the spike transfer probability, characterizes the phenomenon of spike loss. The actual rate of information transfer depends on both quantities as well as on the unknown way of neural coding. The spike loss is the smallest at $1/f^2$ noise (Brownian motion) generated spike train, however, in this case, the widest meaningful bandwidth of information transfer is one order of magnitude less than it is the case of $1/f$ noise. The $1/f$ noise case provides the highest spike propagation rate, though with some compromise in the accuracy of transfer.

IV. STRICTLY PERIODIC INPUT

Let us now consider a periodic main road traffic *before* the junction, i.e., the cars shall arrive at times $t = iG_1v^{-1}$ for $i = 1, 2, \dots$ at the junction J . Here v denotes the unit velocity which shall be set equal 1 for the sake of simplicity. There-with the mean rate of the input is given by the reciprocal of the constant gap size $G = G_1 = G_i$. The number N of cars entering from the side road will then be equal for all G_i , i.e., $N = N_1 = N_2 = \dots$. Hence the mean rate $\nu_{2,3}$ can directly be written down using Eq. (1),

$$\nu_{2,3}(\nu_{1,3}) = \frac{N}{G} = \nu_{1,3} \operatorname{int}\left(\frac{1}{\nu_{1,3}G_0}\right). \quad (7)$$

This equation is the *exact* solution for all $\nu_{1,3}$. To discuss the solution it is useful to introduce three lines subdividing the solution space. The three lines correspond to analytic approximations of the $\operatorname{int}()$ function. The intersections of the solution with the lines are called modes in the following:

- (1) $\operatorname{int}(G/G_0) \approx G/G_0$,
- (2) $\operatorname{int}(G/G_0) \approx G/G_0 - \frac{1}{2}$,
- (3) $\operatorname{int}(G/G_0) \approx G/G_0 - 1$.

(1) *First mode: Best ordered traffic:* The gap sizes of incoming main road cars are minimal for a certain number of side road cars allowed to enter. There is no wasted spacing between consecutive cars which would reduce the overall

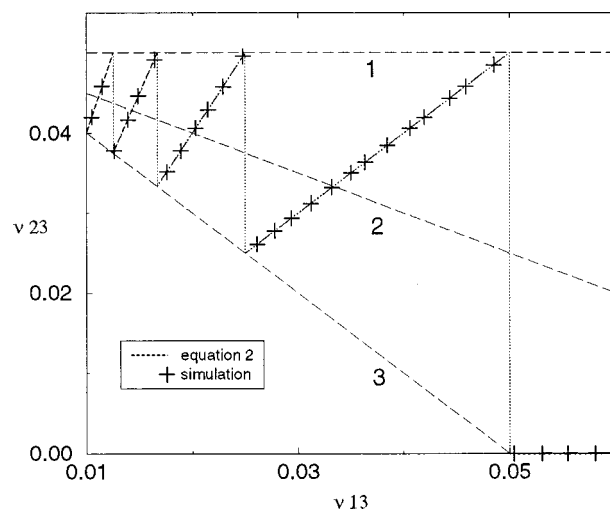


FIG. 5. Side channel mean rate $\nu_{2,3}$ as a response to periodic main channel traffic with mean rate $\nu_{1,3}$ taken from Eq. (7) and simulation (plus signs). The dashed lines mark the three different modes (approximations) of $\nu_{1,3}$: (1) best, (2) medium, and (3) worst ordered traffic.

number of cars passing the junction during a certain time interval and thus impair the overall traffic efficiency.

(2) *Second mode: medium ordered traffic:* This corresponds to an average over all input rates and can be considered as a critical case as will be discussed in Sec. V A.

(3) *Third mode: worst ordered traffic:* The incoming traffic is organized contrarily to 1., i.e., the gap sizes G are infinitesimally smaller than multiples of G_0 , wasting a spacing of (almost) G_0 each time side road cars enter the main road.

Figure 5 illustrates this distinction.

It has been shown in Sec. III that the third mode of periodic traffic is inferior to the traffic triggered by a Poisson process or the zero crossings of $1/f^k$ noise. Moreover it has been assumed that there are “sites” in the main road flow which follow each other strictly by a distance of G_0 . Periodic traffic means that these sites are “filled” by cars in a periodic way. So, the maximal rate is reached when all sites are filled. The next, lower traffic rate is obtained when every second site is filled. Then the next, lower traffic rate is realized, when every third site is filled, etc. This is a sort of “discrete periodic” case. Further on it has to be stated that the straight line which contains the single points of mode 3 is a monotonously decreasing function of the main traffic rate. This monotonous decrease holds on up to the main traffic rate where the cross traffic rate smoothly becomes zero. This is a qualitative behavior one can see in the practice.

Note that the first mode of ordered periodic traffic cannot be exceeded by any other process in the present model. The second mode divides the rate space into two qualitatively different regions. That will be discussed in Sec. V.

V. NOISE MODULATED PERIODIC INPUT

A. Gaussian white noise

The model has been modified to get a more *realistic* description of the arrival times of the incoming main road traffic at the junction. The modification consists of a random

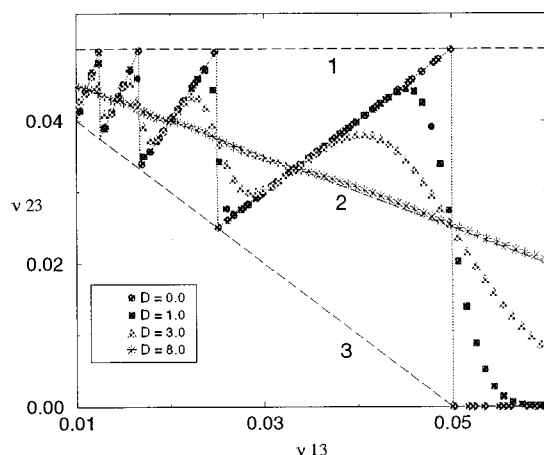


FIG. 6. Simulated effect of noise modulated periodic main channel traffic. An increased noise strength D smoothes the discrete relation between the rates $\nu_{2,3}$ and $\nu_{1,3}$. For rates $\nu_{1,3}$ below the second mode, a noise enhancement for the overall traffic efficiency occurs. In the high noise limit the curve converges to the second mode from both sides.

deviation around the strict periodic position according to a Gaussian distribution with variance D (phase noise). To justify this approach it is worth taking a look at real traffic. Cars pass regulation devices such as traffic lights in an *almost* periodic manner whereas this periodicity gets lost in time due to the individual pattern of behavior or external reasons. $1/f^k$ noise has been observed in traffic flow,³² neuro systems³³ and human coordination.³⁴ Taking the notations of Sec. IV, a periodic traffic with $(\nu_{1,3}, \nu_{2,3})$ lying between modes (2) and (3) can be enhanced by applying the noise in the described way. Note that traffic corresponding to $(\nu_{1,3}, \nu_{2,3})$ between modes (1) and (2) can only be diminished. In this respect, mode (2) is crucial: It separates traffic situations where noise can be beneficial from those where its addition only degrades the system performance. If point $(\nu_{1,3}, \nu_{2,3})$ is lying between modes (2) and (3), speed modulation devices acting randomly as well as random changes of the speed by the driver present possibilities to increase the overall traffic. In case of $G_i \leq G_0$ there would be no side road traffic without the noise at all.

An increase in the variance (or noise strength) D leads to a larger total number of cars passing the junction and thus increases the overall traffic efficiency. The physical reason is that the time-dependent probability distribution of the events (cars) evolves from a periodic configuration of δ peaks to an overlap of Gaussian distributions. For large D the probability distribution becomes uniform, which explains the convergence toward mode (2) for *all* possible input rates below and above mode (2).

The effect can be seen in computer simulations presented in Fig. 6 where the results of the modulation are plotted together with the results obtained for the unmodulated case ($D=0$).

B. Gaussian colored noise

How does the shape of the power spectrum of the applied Gaussian noise influence the increase of the overall traffic efficiency? To answer this question further simulations

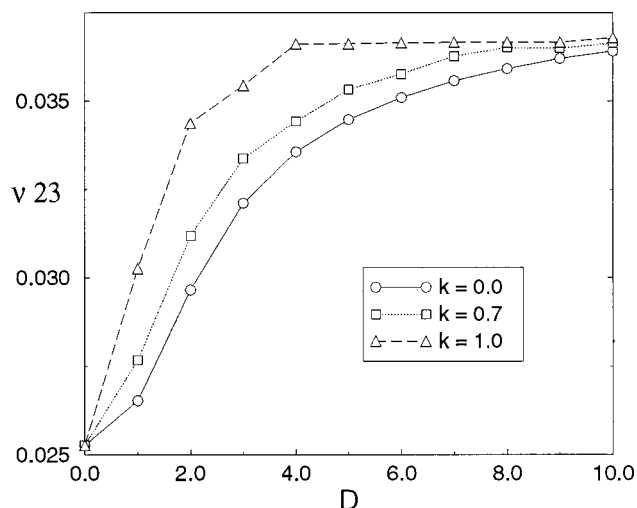


FIG. 7. An increased exponent k in the power spectrum of the applied noise leads to a faster growth of the traffic rate $\nu_{2,3}$ within the investigated parameter range. The input traffic rate has been fixed at a value $\nu_{1,3}=1/39 \approx 0.025$.

has been carried out using long-range correlations in the $1/f^k$ noise with $k>0$. The motivation for introducing $1/f^k$ -like noise has been mentioned previously and corresponds to a widely discussed occurrence of noise with $1/f^k$ or $1/f^k$ -like spectra in various fields.

Figure 7 shows the D -dependent increase of the rate $\nu_{2,3}$ for a fixed value of $\nu_{1,3}$ for different values of the spectral exponent k . A faster convergence toward mode (2) for larger values of k , i.e., for longer correlations within the noise, could be observed.

VI. CONCLUSIONS AND OPEN QUESTIONS

It is shown that noise, i.e., random fluctuations, can enhance the traffic flow at a junction. Periodic arrangements do not always provide the optimal efficiency, but stochastic triggering or modulation can be essential. Further, it is shown that traffic flows generated by $1/f^k$ noises have superior properties over Poissonian and in certain cases over periodic cases too. The best properties are achieved around $k=1$. This fact is in an intriguing coincidence with the general occurrence of $1/f$ -like noise phenomena in neural activity and highway car traffic. Although this model favors this kind of noise in terms of efficiency, the question about the reasons of its appearance in real networks like brains or road systems is still an open one. A complete analysis of the “microscopic” mechanisms in both neural activity and car traffic which should reveal the reasons for the macroscopically observed $1/f$ -like spectra remains to be done.

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